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## Dissertation Defense Abstract

**Two results on divisors on moduli spaces of sheaves on algebraic surfaces:  
generic Strange Duality on abelian surfaces and Nef cones of Hilbert schemes of  
points on surfaces with irregularity zero**

In the first part of this thesis, we consider a special version of Le Potier's strange duality conjecture for sheaves over abelian surfaces, after other two versions were studied in previous literature. In the current setup, the isomorphism involves moduli spaces of sheaves with fixed determinant and fixed determinant of the Fourier-Mukai transform on one side, and moduli spaces where both determinants vary, on the other side. We first establish the isomorphism in rank one using the representation theory of Heisenberg groups. For product abelian surfaces, the isomorphism is then shown to hold for sheaves with fiber degree one via Fourier-Mukai techniques. By degeneration to product geometries, the duality is obtained generically for a large number of numerical types. Finally, it is shown in great generality that the Verlinde sheaves encoding the variation of the spaces of theta functions are locally free over moduli.

In the second part, we discuss general methods for studying the cone of ample divisors on the Hilbert scheme of  $n$  points over a smooth projective surface of irregularity zero. We then use these techniques to compute the cone of ample divisors on the Hilbert scheme of points for several surfaces where the cone was previously unknown. Our examples include families of surfaces of general type and del Pezzo surfaces of degree one. The methods rely on Bridgeland stability and the Positivity Lemma of Bayer and Macrì.